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The theoretical status of quantum chromodynamics

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The foundations of quantum chromodynamics are described and attempts to obtain quantitative results by using both perturbative and non-perturbative techniques are reviewed.

1. INTRODUCTION

I begin by reviewing the original motivation for quantum chromodynamics (QCD) and its numerous qualitative successes. Next I review the U(1) and the θ-parameter problems, which have sometimes been thought lethal for QCD, and explain how they may be circumvented. In the second half of the paper I discuss quantitative investigations of QCD, starting with short-distance physics and working up to long-range properties and the question of confinement. Before summing up I comment briefly on the possibility that quarks are not confined.

2. MOTIVATION FOR QCD

There is excellent evidence, sumarized later, that quarks have a hidden three-valued variable called colour and that there is a global colour symmetry. The force between the tricoloured quarks must be colour dependent; otherwise there would be nine degenerate π mesons with different colours. Furthermore there is convincing evidence that this force is mediated by vector, spin-1, mesons. The only consistent field theory of colour-dependent forces mediated by vector gluons is QCD.

The evidence for colour is as follows.

(i) It resolves the spin-statistics problem, allowing quarks to be put in states that are totally symmetric in spin, space and flavour variables, as is unambiguously required by the very successful quark model for baryons.

(ii) It is required to account for $\Gamma(\pi^{\circ} \rightarrow \gamma\gamma)$ (see below).

(iii) It is required to account for $\sigma(e^+e^- \rightarrow \text{hadrons})$.

(iv) It is needed to exorcise anomalies from $SU(2) \times U(1)$.

Spin-1 exchange is required by the observation of almost exact $SU_L(2) \times SU_R(2)$ chiral symmetry realized in the Nambu–Goldstone mode. In the limit of exact symmetry, in which the weak axial isovector current responsible for π decay and Gamow–Teller transitions is conserved, the following successful predictions can be obtained:

theory	experiment
$M_\pi^2/M_\rho^2 = 0$	0.03
$1+2Mg_A/f_\pi g_{np\pi} = 0$	0.08 ± 0.01
$M_\pi^2 a_{\pi N}^{\frac{1}{2},\frac{3}{2}} = 0.16,\ -0.079$	$0.17 \pm 0.005,\ -0.088 \pm 0.004$
$\lambda_{Ke3}^0 = 0.021$	0.019 ± 0.004
$\Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.87$ eV	7.95 ± 0.55 eV

The second entry in the left-hand column is the Goldberger–Treiman relation, $a^I_{\pi N}$ are pion nucleon scattering lengths, λ^0 is the slope parameter in K_{e3} decay, and the prediction for $\Gamma(\pi^\circ \rightarrow \gamma\gamma)$ includes a factor 3^2 from colour. Clearly the real world is close to a limit in which chiral symmetry is exact. This can only be understood by assuming that gluons have spin-1 and that the bare or current quark masses are small.

3. Qualitative successes of QCD

(a) *Generalities*

In any field theory a distance-dependent 'effective' or 'running' coupling constant $g(r)$ can be defined. Very crudely the r-dependence represents the modification of the one-gluon-exchange Coulomb-type potential between quarks due to vacuum polarization, vertex corrections, etc., i.e. the potential is $\alpha_s(r)/r$, where $\alpha_s \equiv g^2/4\pi$. In contrast to all other field theories, non-Abelian gauge theories such as QCD are asymptotically free (Politzer 1973; Gross & Wilczek 1973), which means that $\alpha_s(r)$ vanishes like $(\ln r)^{-1}$ as $r \rightarrow 0$. Conversely, α_s increases as r increases, perhaps without limit.

It is clear experimentally that the force between quarks has qualitatively this behaviour. It is obviously strong at long distances, and the success of the parton model or impulse approximation for deep inelastic processes shows that it is weak at short distances.

(b) *Chromoelectric forces*

A first guess is that the long-range interquark force has the simplest possible non-trivial colour structure $\boldsymbol{\lambda}\cdot\boldsymbol{\lambda}$, where $\boldsymbol{\lambda}$ represents the eight Gell–Mann matrices. This force would be most attractive for colour singlet states (Nambu 1966, Lipkin 1973, Feynman 1973). In fact the toy Hamiltonian

$$H = C[\Sigma\, \boldsymbol{\lambda}\cdot\boldsymbol{\lambda} + \tfrac{4}{3}(N_q + N_{\bar{q}})],$$

where the sum runs over all qq pairs, gives zero energy for colour singlets and positive energy for all other states. This shows that QCD is potentially capable of explaining the absence of coloured states, although the question of whether they are completely absent can only be answered by detailed dynamical calculations.

(c) *Chromomagnetic forces*

A simple first guess, which turns out to be spectacularly successful, is that the long-range force is spin-independent, spin dependence being due mainly to single-gluon exchange which dominates at short distances in QCD (De Rujula *et al.* 1975). In s wave states, this gives rise to an interaction

$$-\tfrac{2}{3}\pi g^2\, \boldsymbol{\lambda}^1\cdot\boldsymbol{\lambda}^2\, \mu_1 \boldsymbol{\sigma}_1\cdot\mu_2\boldsymbol{\sigma}_2\, \delta(\boldsymbol{r}),$$

where the μ_i are the 'chromomagnetic moments'. Just as the analogous hyperfine interaction lifts the degeneracy of *ortho*- and *para*-positronium, this interaction gives $M_\rho > M_\pi$ and, with antisymmetric spin-space wavefunctions, $M_\Delta > M_N$ and $\langle r^2_{\text{e.m.}}\rangle_n < 0$. The μ_i have dimensions m^{-1} and we would expect $\mu_s \sim m_s^{-1}$ to be less than $\mu_{u,d} \sim m^{-1}_{u,d}$, which would explain why $M_\Sigma > M_\Lambda$. Quantitatively

$$\mu_u/\mu_s = 1 + 3(M_\Sigma - M_\Lambda)/2(M_{\Sigma^*} - M_\Sigma) = 1.60$$
$$= (M_\rho - M_\pi)/(M_{K^*} - M_K) = 1.60.$$

Taking out the charge factors, we find that the $\mu_i^{\mathrm{e.m.}}$, which we would expect to be similar but not identical, satisfy

$$\mu_{\mathrm{u}}^{\mathrm{e.m.}}/\mu_{\mathrm{s}}^{\mathrm{e.m.}} = 1.52.$$

The spin-dependent force abstracted from one-gluon exchange also works very well for baryons with $L \neq 0$ (Isgur 1980).

(d) *Short-range QCD*

The fact that α_s vanishes as $r \rightarrow 0$ or equivalently as the energy $E \rightarrow \infty$ like $(\ln E)^{-1}$, is believed to justify the use of perturbation theory in high energy processes provided one of the following holds.

(i) There are no 'mass singularities', i.e. to all orders in perturbation theory there are no factors of $\ln (E/m)$ where m is the quark mass. Mathematically, factors like $[\alpha_s \ln (E/m)]^n$ spoil perturbation theory at large E. Physically, powers of $\ln m$ are due to configurations in which virtual quarks become almost real and propagate over large distances. Their presence therefore indicates that long distances are important, and a perturbative discussion in terms of quarks and gluons would not be expected to work. Conversely, if there are no terms in $\ln m$ the result may be insensitive to long-distance physics, and a calculation that ignores the way in which quarks and gluons turn into real hadrons might work. An example of a quantity that has no mass singularities is $\sigma(e^+e^- \rightarrow \text{hadrons})$.

(ii) The mass singularities 'factorize'. This means that the dependence on $\ln m$ and on how hadrons are constructed from quarks and leptons can be factored off into an energy-independent factor, which must be taken from experiment, leaving a calculable energy-dependent 'short-distance' factor. This is supposed to happen in all the classic parton processes (Dokshitzer *et al.* 1978, 1980; Llewellyn Smith 1978; Ellis *et al.* 1978; Amati *et al.* 1978*a*, *b*; Mueller 1978; Libby & Sterman 1978*a*, *b*) and for some exclusive quantities such as form factors (Brodsky & Lepage 1979, 1980; Efremov & Radyushkin 1980*a*, *b*; Duncan & Mueller 1980*a*–*c*).

The status of proofs that factorization occurs, and the difficulties of performing quantitative tests of the predictions, will be discussed briefly in § 5*a*. Here are noted the large number of cases in which the predictions work qualitatively at least (reviews of the experimental situation can be found in the *Proceedings of the 20th Int. Conf. on High Energy Physics*, Madison, Wisconsin (ed. L. Durand & L. G. Pondrom) (New York: American Institute of Physics (1980)) and in the report by D. H. Perkins in this symposium):

(i) Deep inelastic structure functions: the naive parton model is a good first approximation and the scaling violations predicted by QCD are observed.

(ii) $\sigma(e^+e^- \rightarrow \text{hadrons})$ scales but unfortunately the data are not accurate enough to show the small deviations from the naive parton model predicted by QCD.

(iii) The three-jet events in e^+e^- annihilation predicted by QCD have been seen with the expected features.

(iv) The cross section for $p(\pi)p \rightarrow \mu\bar{\mu}X$ is well described by the naive parton model except that the absolute value of σ is too big by a factor of $O(2)$, as expected on the basis of higher-order QCD calculations (Altarelli *et al.* 1978*a*, *b*; 1979).

(v) The transverse momentum of the μ pair in $pp \rightarrow \mu\bar{\mu}X$ increases with s as predicted by QCD.

(vi) The transverse momentum of the jets observed in leptoproduction increases with W as predicted by QCD.

(vii) The structure function of the photon is well described by the parton model although the data are not yet accurate enough to show the modifications predicted by QCD.

(viii) The cross section for the production of large-p_T particles in $\gamma\gamma$ collisions has the expected form.

(ix) The production of large-p_T photons in pp collisions has the expected form.

(x) Leptoproduction of charmed particles is well described by the QCD-based gluon fusion model.

The only case where high hopes of testing QCD have been disappointed is in the production of large-p_T particles in pp collisions. However, the problem is that these processes are very complex; the data are not inconsistent with QCD. Although there is no single definitive test, QCD works quantitatively as well as could be expected, and the fact that it survives so many qualitative tests is impressive.

4. Possible problems

(a) *The* U(1) *problems*

In QCD the $I = 1$ axial current $J_\mu^{5, I=1}$ is conserved in the limit $m_{u,d} \to 0$ and consequently $M_\pi \to 0$, as discussed in § 2. It would seem naively that the $I = 0$ current

$$J_\mu^5 = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d$$

would also be conserved in this limit, giving rise to a massless isoscalar Goldstone boson (Glashow 1968). With $m_{u,d} \neq 0$, standard current algebra methods predict $M_{I=0} < \sqrt{3}M_\pi$ for its mass (Weinberg 1975). This is the first U(1) problem.

It is now known that anomalies associated with J_μ^5 provide a possible loophole in the arguments that lead to this catastrophic prediction (for detailed reviews see Crewther (1979*a*, *b*) and Peccei (1980)). When calculating the matrix elements of currents, a regularization scheme is needed to render finite the contributions of individual diagrams. Because of Ward identities, the matrix elements will be finite and unique when the regulator is removed provided it respects the symmetries of the theory. The case of J_μ^5 is anomalous because it is impossible to devise a procedure that respects the symmetries associated with both vector and axial currents for certain diagrams (Adler 1970; Jackiw 1972). The simplest example is the contribution of the fermion triangle diagram to $\langle 0| J_\mu^5 V_\alpha V'_\beta |0\rangle$ where V and V' are vector currents. Various J_μ^5 can therefore be defined depending on the regularization procedure.

The current $J_\mu^{5(V)}$ defined by insisting on vector Ward identities has an anomalous divergence

$$\partial^\mu J_\mu^{5(V)} = 4(g^2/32\pi^2)\, G_{\mu\nu}\cdot\tilde{G}_{\mu\nu} + 2\mathrm{i}m_u\,\bar{u}\gamma_5 u + 2\mathrm{i}m_d\,\bar{d}\gamma_5 d,$$

where $G_{\mu\nu}$ is the covariant field tensor and

$$\tilde{G}_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$$

is its dual. However, we can also insist on the axial identities and define a current $J_\mu^{5(A)}$ whose divergence has the naive value that vanishes as $m_{u,d} \to 0$. The associated charge $Q^5 = \int J_\mu^{5(A)}\,\mathrm{d}^3x$ generates an exact U(1) symmetry for $m_{u,d} = 0$. The vacuum cannot be U(1)-invariant since

$$\langle 0| [Q^5, \bar{u}_L U_R] |0\rangle = -2\langle 0| \bar{u}_L U_R |0\rangle$$

does not vanish, given that isovector chiral symmetry is spontaneously broken. It follows that U(1) must be spontaneously broken also, and, in covariant gauges at least, there must be a massless isoscalar particle coupled to $J_\mu^{5(A)}$ in the symmetry limit.

However, because of the anomaly the symmetry current $J_\mu^{5(A)}$ is gauge-dependent. Kogut &

Susskind (1975) pointed out that this provides a possible resolution of the U(1) problem since it is conceivable that the massless isoscalar, which must couple to $J_\mu^{5(A)}$ in covariant gauges, does not generate poles in gauge-invariant quantities. They showed that an analogous situation occurs in the two-dimensional Schwinger model. The gauge dependence of $J_\mu^{5(A)}$ is demonstrated explicitly by the fact that the gauge-variant current

$$K_\mu \equiv (g^2/32\pi^2)\,\epsilon_{\mu\nu\alpha\beta} A_\nu \cdot (G_{\alpha\beta} - \tfrac{1}{3} g A_\alpha \times A_\beta)$$

satisfies

$$\partial^\mu K_\mu = (g^2/32\pi^2)\, G_{\mu\nu}\cdot \tilde{G}_{\mu\nu}$$

so that $J_\mu^{5(A)}$ is related to the gauge-invariant current $J_\mu^{5(V)}$ by

$$J_\mu^{5(V)} = J_\mu^{5(A)} + 4K_\mu$$

up to terms whose divergence vanishes identically.

A curious aspect of this possible solution of the U(1) problem is revealed by considering

$$\int d^4x\, \partial^\mu \langle 0|\, T(J_\mu^{5(A)}(x)\, \partial^\nu J_\nu^{5(A)}(0))\, |0\rangle$$
$$= \int d^4x \langle 0|\, T(\partial^\mu J_\mu^{5(A)}(x) \partial^\nu J_\nu^{5(A)}(0))\, |0\rangle + \langle 0|\, [Q_5, \partial^\nu J_\nu^{5(A)}(0)]\, |0\rangle.$$

In the analogous expression for $J_\mu^{5,\,I=1}$ the left-hand side is zero for $m_q \neq 0$ since there are no massless isovectors. On the right-hand side the first term is apparently $O(m_q^2)$ but the pion pole contribution $f_\pi^2 M_\pi^2$ must cancel the last term which is proportional to $m_u \langle \bar{u}u\rangle + m_d \langle \bar{d}d\rangle$; the standard interpretation is that $M_\pi^2 = O(m_q)$, and f_π and $\langle \bar{q}q\rangle$ are $O[(m_q)^\circ]$. In the equation displayed above the second term on the right-hand side is again proportional to $m_u \langle \bar{u}u\rangle + m_d \langle dd\rangle$ and must be cancelled by a pole in the first term with $M_{I=0} = O(m_q)$ is the left-hand side vanishes. This argument leads to the disastrous bound $M_{I=0} < \sqrt{3}\, M_\pi$. It can only be avoided if the $I = 0$ particle coupled to $J_\mu^{5(A)}$ is massless *even for* $m_q \neq 0$ so that the left-hand side need not vanish.

This necessity is made more palatable by the work of 't Hooft (1976*a*, *b*). He showed that in the dilute instanton gas approximation $\langle 0|\, \Sigma_f \bar{q}_f q_f\, |0\rangle \neq 0$, so the U(1) symmetry is spontaneously broken, and $\partial^\mu \langle a|\, K_\mu\, |b\rangle \neq 0$ for zero momentum transfer so there is a massless particle coupled to K_μ and $J_\mu^{5(A)}$ which, it turns out, does not couple to $J_\mu^{5(V)}$ (instantons are discussed briefly in §4*b*; they provide the necessary vacuum degeneracy for spontaneous U(1) breaking as the θ vacua are degenerate for $m_q = 0$). This is very encouraging but, as repeatedly emphasized by Crewther (1978, 1979*a*, *b*), it does not prove that the U(1) problem is solved since the dilute gas approximation is very remote from the real world. Indeed, the compactified boundary conditions that prevent massless particles from coupling to the gauge-invariant current $J_\mu^{5(V)}$ also prevent spontaneous breaking of isovector chiral symmetry.

The Kogut–Susskind mechanism would also provide a solution of the second U(1) problem. It was shown long ago by Sutherland (1966) using standard current algebra techniques that virtual photon exchange gives a vanishing amplitude for $\eta \to 3\pi$ in the chiral limit. This decay must therefore be attributed to the isospin-violating term

$$H' = \tfrac{1}{2}(m_u - m_d)\,(\bar{u}u - \bar{d}d)$$

in the Hamiltonian density. The leading terms in chiral perturbation theory give (see, for example, Bell & Sutherland 1968)

$$\langle 3\pi|\, H'\, |\eta\rangle \xrightarrow{q_{\pi^0}\to 0} (m_u - m_d)\, A/\sqrt{2}\, f_\pi^2,$$

where

$$A = \langle \pi^0\pi^0 | m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d | \eta \rangle,$$

and the amplitudes are given by

$$A(\eta \to 3\pi^0) = (m_u - m_d)\, A/\surd 2 f_\pi^2,$$

$$A(\eta \to \pi^+\pi^-\pi^0) = [(m_u - m_d)/\surd 2 f_\pi^2]\, A(1 - 2E_0/M_\eta),$$

in good agreement with experiment if A is treated as an arbitrary parameter. However

$$2iA = \langle \pi\pi | \partial^\mu J_\mu^{5(A)}(0) | \eta \rangle,$$

where the dipion and η have equal momenta and energies, which vanishes unless there are zero-mass particles coupled to $J_\mu^{5(A)}$. Thus $\eta \to 3\pi$ would be forbidden in the chiral limit if it were not for the Kogut–Susskind pole (Brandt & Preparata 1970). In fact, if we assume that the U(1) problem is solved, further standard current algebra manipulations give (Weinberg 1975)

$$A(\eta \to 3\pi^0) = [(m_u - m_d)/2m_s]\,(8M_K^2/3\surd 3 f_\pi^2).$$

A fit to the observed rate yields (Langacker & Pagels 1979)

$$(m_d - m_u)/2m_s = 0.0145 \pm 0.0015.$$

For comparison, to lowest order in chiral and SU(3) symmetry breaking

$$\frac{m_d}{m_u} = \frac{(K^0)^2 - (K^+)^2 + (\pi^+)^2}{(2\pi^0)^2 + (K^+)^2 - (K^0)^2 - (\pi^+)^2} = 1.80,$$

$$(m_d + m_u)/2m_s = M_\pi^2/2M_K^2 = 0.038,$$

$$(m_d - m_u)/2m_s = 0.011.$$

With the inclusion of higher-order corrections, an analysis of meson (baryon) masses gives (Langacker & Pagels 1979)

$$(m_d - m_u)/2m_s = 0.0175 \quad (0.0105 \pm 0.003),$$

in satisfactory agreement with the value obtained from η decay.

It cannot at present be proved that the U(1) problems are solved by the Kogut–Susskind mechanism, although presumably this must be the case if QCD is correct. However, this solution is believed to be theoretically and phenomenologically viable. In particular, work by Witten (1979*b*), Crewther (1980) and others has shown that it is compatible with the necessary conditions for solving the U(1) problem, derived by Crewther (1979*a*), and that various potential paradoxes are avoided. This has been demonstrated explicitly by the construction of physically acceptable effective Lagrangians that satisfy all the Ward identities in the limit of large N_c, where N_c is the number of colours (Di Vecchia & Veneziano 1980; Rosenzweig *et al.* 1980; Arnowitt & Nath 1980; Witten 1980; Kawarbayashi & Ohta 1980).

It turns out that $M_\eta^2 = O(1/N_c)$ (Witten 1979*a*, *b*). This is not unexpected since, as discussed below, $g^2 = O(1/N_c)$ so that for $N_c \to \infty$ the anomaly vanishes and $J_\mu^{5(V)}$ must couple to a physical massless boson as it is conserved. This is consistent with the old picture (De Rujula *et al.* 1975; Isgur 1976) which attributes pseudoscalar masses to normal $m_q \langle \bar{q}q \rangle$ contributions plus mixing or 'gluon annihilation' terms in the isosinglet sector. One parameter is needed to describe the anomaly or annihilation term, which can be fixed by $M_\eta^2 + M_{\eta'}^2$. The parameter $\Delta = M_{\eta'}^2 - M_\eta^2$, and the octet singlet mixing angle are then predicted to be

$$\Delta = 0.72\,\mathrm{GeV}^2, \quad \phi = 18°,$$

in reasonable agreement with the experimental values of 0.62 GeV² and 10°. The agreement can be improved by including non-leading terms of $O(m_q/N_c)$ (Di Vecchia *et al.* 1981).

The effective Lagrangian leads to other predictions for η' decay and, with additional assumptions, for ψ decay (Di Vecchia *et al.* 1981; Dyakonov & Eides 1981; Kawarbayashi & Ohta 1980; Milton *et al.* 1980). The most reliable is for $\eta' \rightarrow 3\pi$ but unfortunately only an experimental limit is available.

(*b*) *The θ-parameter*

The QCD Lagrangian contains a term

$$\theta(g^2/32\pi^2)\, G_{\mu\nu}\cdot\tilde{G}_{\mu\nu},$$

where θ is an arbitrary parameter. Its contribution to the action can be written as a surface integral

$$\theta\int \partial^\mu K_\mu \,\mathrm{d}^4x = \theta\int K_\mu \,\mathrm{d}\sigma_\mu,$$

where K_μ is the current defined in §4*a*. Normally such surface terms are discarded since the boundary condition that fields vanish at infinity is assumed. Even if this is the case, which is by no means obvious in QCD, the field tensor $G_{\mu\nu}$ can vanish like r^{-2} or faster but A_μ can tend to a pure gauge configuration such that $\int K_\mu \,\mathrm{d}\sigma^\mu \neq 0$. If we perturb about $A_\mu = 0$ these configurations are not seen in any finite order. However, the existence of instanton solutions of the Euclidean equations of motion (Belavin *et al* 1975) shows that they are separated by a finite potential barrier and can be reached by tunnelling with amplitude proportional to e^{-c/g^2} (Jackiw & Rebbi 1976; Callan *et al.* 1976). It follows that the θ-term cannot be discarded.

Before discussing the consequences, I shall digress briefly to discuss the role of instantons. For very small g, corresponding to very small distances, tunnelling is negligible and QCD perturbation theory can presumably be used in appropriate circumstances. For some intermediate values of g it may make sense in some cases to introduce the so called 'θ-vacuum', which is a coherent superposition of pure gauge states with $\int K_\mu \,\mathrm{d}\sigma^\mu \neq 0$, and to continue to use perturbation theory with allowance for tunnelling in the dilute instanton gas approximation. This produces a sudden rapid increase in g, departing from ordinary perturbation theory, as r is increased (Callan *et al.* 1980) which is expected on phenomenological grounds for $r = 0$ (0.5 fm) and is seen in the lattice calculations discussed below. For large distances it is not clear that instantons and the pure gauge θ-vacuum have any direct relevance, although they remind us that the QCD vacuum must be very complicated.

The θ-term obviously violates P and therefore also T, since it clearly conserves C. In general, therefore, it will produce a neutron dipole moment and standard current algebra techniques show that the present limit requires $\theta < 1.5\times10^{-9}$ (Baluni 1979; Crewther *et al.* 1979). The θ-problem is to provide a rationale for this result given that θ is a strong interaction parameter which *a priori* we would expect to be $O(1)$. It is tempting to argue that $\theta = 0$ is a natural simple value. However, in general higher-order weak interactions introduce divergent CP-violating effects that can only be cancelled by introducing a divergent cut-off-dependent θ-term in the

Lagrangian. It seems usually to be thought that the finite part of θ is then completely arbitrary. There are various possible ways out of this dilemma.

(i) CP-violation is due to 'soft' terms, the relevant operators having dimension less than three; there are then no divergences that need to be cancelled by the θ-term. No convincing models of this sort are known (Senjanovic 1980).

(ii) It is not obvious that when perturbation theory requires a divergent θ-term the finite part is completely arbitrary. Presumably the cut-off Λ expresses a modification of the physics. Although terms involving $g^n \ln \Lambda$ are incalculable, we might be surprised if $\ln \Lambda$ were as big as, say, 100, and much smaller numbers are not obviously unnatural. In fact it turns out that with Λ equal to the Planck mass the induced θ is much smaller than 10^{-9} in the minimal standard model but is greater than or of order 10^{-12} in models with additional CP-violation introduced to explain the cosmic value of n_B/n_γ (Ellis *et al.* 1981).

(iii) If one of the quark masses (presumably m_u) is zero, so that there is an exact U(1) symmetry, the Ward identities show that a U(1) transformation is equivalent to a change of θ, assuming that the U(1) problem is solved (in fact θ effectively labels the degenerate vacua associated with spontaneous U(1) symmetry breaking). In this case the S-matrix is independent of θ (Peccei & Quinn 1977*a*, *b*). However, current algebra calculations show that it is extremely unlikely that $m_u = 0$ (Langacker & Pagels 1979).

(iv) By increasing the number of Higgs mesons it is possible to arrange that even if $m_{u,d} \neq 0$ there is a global $U(1)_{PQ}$ symmetry, whose effects are equivalent to changing θ, which can then be rotated to zero (Peccei & Quinn 1977*a*, *b*). However, one of the Higgs mesons (the axion) plays the role of the Goldstone boson for $U(1)_{PQ}$ and is massless at the classical level (Weinberg 1978; Wilczek 1978). Because of anomalies the symmetry is not really exact and the axion acquires a small mass. The existence of this object seemed to be ruled out some years ago but a recent experiment has reported evidence for an axion-like object (Faissner *et al.* 1981).

5. Quantitative QCD

(*a*) *Short distances*

More work is needed on two aspects of the foundations of perturbative QCD.

(i) *Infrared divergences and effects due to soft gluons.*† It has been proved that, order by order in perturbation theory, soft gluons do not spoil factorization in processes with no hadrons in the initial state (Collins & Sterman 1981). However, there are difficulties in extending the proof convincingly to processes with initial hadrons. Indeed, if there are coloured particles in the initial state it is known that infrared divergences do not cancel in some 'higher-twist' terms, which are $O(m^2/E^2)$ relative to the leading terms (Doria *et al.* 1980; Andrasi *et al.* 1981; Di'Lieto *et al.* 1981).

(ii) *The influence of non-perturbative effects such as binding on factorization.* A possible puzzle is that if factorization holds then, for $s \to \infty$ with Q^2/s fixed, $pA \to \mu\bar{\mu}X$ must have the same dependence on the nuclear number A as $eA \to eX$: presumably A^1. This requires hard partons in the proton to reach the back of the target nucleus with zero inelasticity. It is easy to understand that this could be a good approximation as simple estimates suggest a long mean free path for partons in nuclei (and the observation that in fact $\sigma(pA \to \mu\bar{\mu}X) \sim A^\alpha$ with $\alpha = 1 \pm 0(0.1)$ for large Q^2 requires this; for a review see Matthiae (1980)). However, factorization requires that it be

† See note added in proof (p. 20).

exactly true up to $O(m^2/E^2)$. If this is the case, it would be interesting to understand physically how it comes about.

I have already briefly reviewed the many qualitative tests of QCD. More detailed tests are difficult for two reasons.

(i) To see the predicted energy dependence requires a large range of E, but terms that are higher order in α_s and $O(m^2/E^2)$ are large at modest energy and mask the predicted variation.

(ii) The coefficients of α_s^n in higher-order terms are large in many cases, and the interpretation is complicated by the fact that they depend on the scheme used to definite α_s and the choice of variables. This has been widely discussed (see, for example, Llewellyn Smith 1980; Roberts 1981).

More work is clearly needed on the following:

(i) The best choice of scheme and of variables;

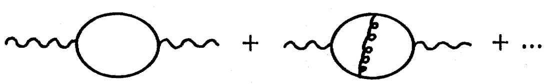
(ii) the summation of 'large' corrections to improve the convergence of perturbation theory;

(iii) summation of double logarithms which may allow perturbation theory to be used in new régimes (see, for example, Collins & Soper 1981);

(iv) the $O(m^2/E^2)$ higher-twist effects.

(*b*) *Intermediate distances*

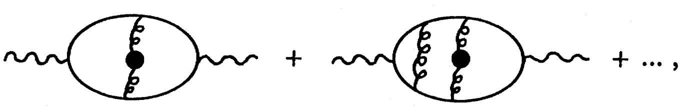
The ITEP group have initiated a programme that may make it possible to extend the use of perturbation theory to lower energies or longer distances by incorporating phenomenologically some information that cannot be obtained perturbatively (Shifman *et al.* 1979*a*, *b*, *c*; Eidelman *et al.* 1979). Before describing the details, I shall illustrate the idea very crudely for the electromagnetic current. The quantity $R_{\bar{e}e}$ averaged over a range ΔE is controlled by $\langle 0|\, J_\mu^{\text{e.m.}}(x)\, J_\mu^{\text{e.m.}}(0)\,|0\rangle$ for $x = O(1/\Delta E)$. For small x the current correlation function can be calculated perturbatively. The usual diagrams



contribute terms of $O(1)$ to $\langle R_{ee}\rangle$, which can be calculated as a power series in $\alpha_s(\Delta E)$, with power corrections of $O(m_q^2/\Delta E^2)$ which are negligible for light quarks. Non-perturbative effects can introduce other power corrections involving different scales that cannot be seen in any order of perturbation theory. For example, chiral symmetry-breaking introduces $m_q\langle 0|\, qq\,|0\rangle/\Delta E^4$-terms but they will also be unimportant unless ΔE is very small. Larger effects might occur if

$$\kappa \equiv (\alpha_s/\pi)\,\langle 0|\, G_{\mu\nu}\cdot G_{\mu\nu}\,|0\rangle \neq 0.$$

The $O(\kappa/\Delta E^4)$ contributions to $\langle R_{ee}\rangle$ can be calculated to arbitrary order in α_s, κ being treated as a parameter, from diagrams such as



where the 'QCD vacuum blob' which represents κ emits zero-momentum gluons. The idea of the ITEP group is that when these terms are included ΔE can be made quite small, perhaps even small enough to include single resonances whose properties can be described in terms of the parameter κ.

More explicitly, if

$$-3q^2\pi(q^2) \equiv \mathrm{i}\int \mathrm{e}^{\mathrm{i}qx}\,\langle 0|\, T[J_\mu(x)\, J_\mu(0)]\,|0\rangle\, \mathrm{d}^4x$$

then the operator product expansion can be used to write

$$\frac{\mathrm{d}\pi}{\mathrm{d}s} = \frac{1}{12\pi}\int \frac{R(s')}{(s'-s)^2}\,\mathrm{d}s' = C(s, m_q^2)\,\langle 0|\,I\,|0\rangle + C'(s, m_q^2)\,\kappa/s + \ldots,$$

where C and C' can be calculated perturbatively. The operator product expansion is abstracted from perturbation theory, and its use to describe phenomena that are not seen in any finite order might be questioned. In the dilute instanton gas approximation the operator product expansion does indeed fail but only for contributions of $O(\Delta E^{-p})$ with $p \geqslant 11$, which are neglected. The equation displayed above can be studied in many ways (and for many different currents). The simpliest procedure is to study $\mathrm{d}^n\pi/\mathrm{d}s^n$ for spacelike s. Large n and small $|s|$ enhance the resonance region in the integral but they also increase the importance of the neglected power corrections and the higher orders in $\alpha_s(s)$ in the theoretical expression. The art, then, is to find circumstances in which there is a range of s and n such that only a few resonances contribute but the theory is under control.

The best case is probably $J = \bar{c}\Gamma c$ in which the large value of M_c allows us to put $s = 0$ safely, giving

$$\frac{n!}{12\pi}\int \frac{R_{cc}(s')\,\mathrm{d}s'}{(s')^{n+1}} = \frac{1}{M_c^{2n}}\left\{f_n[\alpha_s(M_c)] + \frac{f_n'[\alpha_s(M_c)]}{M_c^4}\,\kappa + O(1/M_c^6)\right\}.$$

First let $\Gamma = \gamma_\mu$ in which case triplet s states contribute to R. For orientation consider the $n = 3, 4$ sum rules with $f_n = f_n(0)$ and $f_n'\kappa = 0$, and assume that only the ψ contributes to the integral. The sum rules then predict $\Gamma_{\psi\to\bar{e}e} = 5\,\mathrm{keV}$ (the experimental value is $4.4 \pm 1.4\,\mathrm{keV}$)! For larger n it will not be good approximation to neglect $f_n'\kappa$ unless κ is very small as f_n'/f_n increases rapidly with n. The ITEP group claim that the data require $\kappa = 0.01\,\mathrm{GeV}^4$ and it turns out that the conclusion that $\kappa \neq 0$ is quite stable against experimental and theoretical uncertainties (Guberina *et al.* 1980). This is an important result. It demonstrates the non-trivial nature of the QCD vacuum and provides a point of comparison between theory and experiment. For example, the lattice gauge calculations described later can be used to calculate κ with results in reasonable agreement with the measured value (Banks *et al.* 1981; Kripfganz 1981).

By choosing Γ it is possible to make predictions also for the singlet s states and p states. Not only do the sum rules predict the correct order for the states but they also allow a satisfactory quantitative fit (Reinders *et al.* 1980*a–c*, 1981) with a comparable number of parameters to potential models (see, for example, Eichten 1980). This gives confidence in the approach but for a real check we should like to consider the Υ system. Unfortunately, however, the errors are too large for us to be sure that the sum rules work, and in any case the κ contribution is unimportant since it is multiplied by M_b^{-4} (Guberina *et al.* 1980).

The ITEP group have extended their programme to light quarks. They consider (Shifman *et al.* 1979*a–c*; see also Narison & de Rafael 1981)

$$\int \mathrm{e}^{-s/M^2} R(s)\,\mathrm{d}s = \lim_{\substack{s,\,n\to\infty \\ M^2 = s/n \text{ fixed}}} \frac{s^{n+1}}{n!}\frac{\mathrm{d}^n\pi}{\mathrm{d}s^n},$$

which is predicted to be

$$\tfrac{3}{2}M^2\left[1 + \frac{\alpha_s(M^2)}{\pi} + \frac{4\pi^2}{M^4}\langle 0|\,m_u\bar{u}u + m_d\bar{d}d\,|0\rangle + \frac{\pi^2\kappa}{3M^4} + O(1/M^6)\right],$$

for $I = 1$ currents. Small M^2 is needed to amplify resonances but it increases the unknown contributions on the right so again a compromise is needed. The ITEP group use $M = M_\rho^2$, arguing that $\alpha_s(M_\rho^2) \approx 0.5$ is not too large to spoil perturbation theory completely and that, since

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the M_ρ^{-4} and M_ρ^{-6} terms are small, neglected terms may be negligible. This is questionable, however, as the M_ρ^{-6} term is 50 % bigger than the M_ρ^{-4} term. For orientation, with just the ρ on the left and $1.5M_\rho^2$ on the right being retained, the sum rule gives $g_\rho^2/4\pi = 2.3$ (experiment gives 2.36 ± 0.18!) Keeping the known terms and varying M^2 in a region around M_ρ^2 the ITEP group calculate $M_\rho^2 = 0.6\,\mathrm{GeV}^2$! It is hard to know what significance to attribute to these gratifying results or to the successful results obtained by studying axial currents (Shifman *et al.* 1979*a–c*).

I should also mention a very recent paper by Ioffe (1981) who follows the same procedure for a current built of three quarks, saturating the sum rules with the nucleon or Δ. This provides an explicit realization of the fact that $M_N^3 = C\langle 0|\bar{q}q|0\rangle$ in the chiral limit, and Ioffe obtains $M_N \approx 1\,\mathrm{GeV}$ which is non-trivial since $\langle 0|\bar{q}q|0\rangle \approx -(240\,\mathrm{MeV})^3$. Furthermore, he obtains $M_\Delta \approx 1.4\,\mathrm{GeV}$. Again the significance and the errors introduced by the various approximations are hard to assess. In particular it is puzzling that $M_\Delta - M_N$ does not seem to be dominated by one-gluon exchange as it is in successful QCD-based models of baryon spectroscopy.

In conclusion, the evaluation of κ from the cc system is important. However, it is unclear whether the sum rules will be useful for studying systems of heavier quarks. The applications to light quarks are intriguing but there is no control over the corrections at present and the degree to which the results are fortuitous is unclear (for studies of this question in soluble models see Bell & Bertlemann (1980, 1981), and Bradley *et al.* (1981)).

(*c*) *Long-range QCD*

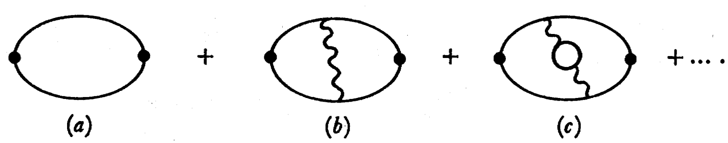
In treating light hadrons it is presumably a good approximation to neglect all but the u and d quarks and work in the limit of exact chiral symmetry $m_{u,d} = 0$. There are then no parameters with dimensions. To begin we can introduce a unit of mass μ at which $\alpha_s(\mu) \equiv 1$. The proton mass will presumably be generated dynamically. In principle M_p/μ and $\alpha_s(M_p)$ are calculable and there are *no* parameters in the theory (except θ). This wonderful fact immediately raises the unsolved question of why m_u, m_d and heavy quark masses, which are introduced as parameters from outside QCD, are within an order of magnitude of the mass scale Λ of QCD.

It also means that there is apparently no parameter in which to expand about an exact solution. A possible procedure ('t Hooft 1974; Witten 1979*c*; Coleman 1980) is to generalize the colour symmetry SU(3) to SU(N), to try to solve the theory for $N \to \infty$ with g^2N fixed, and to perturb in $1/N$. As $N \to \infty$ the theory simplifies and only planar diagrams with quarks at the edges survive, so there is some hope that it may be soluble (as it is in two dimensions). Furthermore, with the assumptions that confinement survives for $N \to \infty$ and that the limit makes sense, it can be shown that the theory exhibits many features of the real world. Thus the large-N limit can be invoked to explain these features and conversely they provide empirical evidence that it is a good approximation. Examples are given below ('t Hooft 1974; Witten 1979*c*).

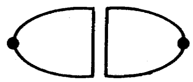
(i) If a meson is represented as

$$|q\bar{q}\,\text{glue}\rangle + |q\bar{q}q\bar{q}\,\text{glue}\rangle + \ldots,$$

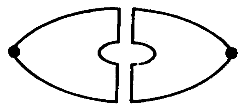
the second term is reduced by $1/N$ relative to the first, and terms with more $q\bar{q}$ pairs are reduced by higher powers. This can be seen by examining the current–current correlation function for two colourless currents:



For large N, the colour properties of the gluon are the same as $\bar{q}_i q_f$ since the term that makes A^{μ}_{if} traceless in colour is $O(1/N)$. The colour flow in diagram (*b*) can therefore be represented as

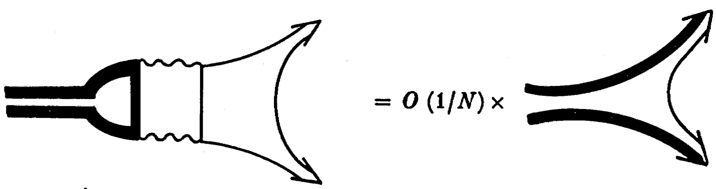


There is a factor N for each closed loop in such a diagram and with $g^2 \sim 1/N$ it behaves like N, as does diagram (*a*) (the behaviour $g^2 \sim 1/N$ is needed to prevent higher orders blowing up faster and faster in this and in other cases such as corrections to the gluon propagator). Diagram (*c*), which can be represented as:



is $O(1)$ and can be neglected compared with (*a*) and (*b*) for $N \rightarrow \infty$. In fact diagrams with extra quarks are always non-leading, and the meson poles in the correlation function contain just $|q\bar{q}\ \text{glue}\rangle$. There is some evidence that the $q\bar{q}$ 'sea' in mesons is small, as it certainly is in baryons. The dominance of $|q\bar{q}\ \text{glue}\rangle$ accounts for the fact that it is much more profitable to picture, say, the ρ meson as a $q\bar{q}$ state than a ππ bound state. It also accounts for the absence of exotics.

(ii) Zweig's rule is exact to leading order as can easily be seen for:



where thick (thin) lines represent heavy (light) quarks.

(iii) Confinement being assumed, the meson spectrum must consist of an infinite number of stable states in leading order. An infinite number is needed for consistency with the logarithmic increase of the real part of the vacuum polarization tensor as $q^2 \rightarrow -\infty$, which is predicted by QCD. Stability follows from the absence of a $q\bar{q}$ sea. This is reminiscent of the dual model, long thought to be a reasonable first approximation to nature, and is consistent with the observation of linearly rising Regge trajectories. It turns out that meson decay widths satisfy

$$\Gamma(\rightarrow \text{n mesons}) = O(1/N^{n-1})$$

which is consistent with the fact that two-body decays seem to dominate, for example

$$\Gamma(\text{B} \rightarrow \omega\pi) \gg \Gamma(\text{B} \rightarrow 4\pi).$$

(iv) Glueball states are decoupled from mesons to leading order. The lighter states are predicted to be narrow since the widths are $O(N^{-2})$.

These results are encouraging but unfortunately there has been no real progress in solving QCD for $N \rightarrow \infty$ (see, however, Witten 1979*d*)) or showing that this limit makes sense mathematically. Furthermore it is not clear that the $1/N$-expansion is useful for studying baryons, which consist of N quarks in SU(N) (see, however, Witten 1979*c*).

A very different approach that has been vigorously pursued in the last two years is to study QCD on a discrete space or space–time lattice. Presumably a lattice of spacing a will give a good approximation to the continuum theory for phenomena on a scale L provided $L \gg a$ and $a \ll 1$ fm. The motivation for introducing a lattice is as follows.

(i) It provides an ultraviolet cut-off divorced from perturbation theory by excluding momenta above a^{-1}. It also introduces a scale at which the coupling constant $g(a)$ can be defined.

(ii) The cut-off on high momenta bounds the kinetic part of the Hamiltonian which can therefore be treated as a perturbation in a strong coupling expansion.

(iii) With a finite lattice, which is presumably acceptable provided it is much greater than both L and 1 fm, there are only a finite number of variables. It is therefore possible to study the energy spectrum, correlation functions, etc. by calculating Feynman path integrals using a computer for a Euclidean version of the theory formulated in imaginary time (for an overview with references see Parisi (1980)).

In both Abelian and non-Abelian lattice gauge theories confinement can be demonstrated for large $g(a)$ by using the strong-coupling expansion (Wilson 1974). Infinitely heavy Q and $\bar{\text{Q}}$ are placed on the lattice at separation R, light quarks being ignored as in all the calculations described below. As R is increased with a and $g(a)$ fixed, the flux lines joining Q and $\bar{\text{Q}}$ tend to form a narrow tube which follows the shortest path, and the energy increases linearly with R. This is consistent with potential models of Q$\bar{\text{Q}}$ systems and linearly rising Regge trajectories, which give $\sqrt{K} \approx 0.34$ GeV and 0.4 GeV respectively for the 'string tension' K defined by

$$E(R) \xrightarrow{R\to\infty} \tfrac{1}{2}\pi KR.$$

The function Ka^2 is a calculable function of $g^2(a)$, which allows $g^2(a)$ to be fixed in terms of K; to leading order in the strong-coupling expansion† $g^2(a) = \frac{1}{3}\,e^{Ka^2}$. The question is whether this behaviour goes smoothly into $g^2(a) \sim 1/(\ln a^{-1})$ for $a \to 0$, i.e. whether confinement and asymptotic freedom coexist in a single phase of QCD.

Generally $g(a)$ can be defined implicitly by $Ma = f[g(a)]$ where M is an observable with dimensions of mass; for example M could be the mass of the lightest glueball or, in a confining phase, $\sqrt{K}$. Which observable M is held fixed as a varies does not matter since other quantities M' become independent of a for $a \to 0$, with corrections of $O(a^2/L^2) + O(m^2a^2)$ where L is the scale of interest and m a typical hadronic mass scale. In terms of the variable $\beta \equiv 6/g^2(a)$ ($\beta \equiv 2N/g^2(a)$ for SU(N)), we know that Ma decreases as β increases in the strong-coupling limit ($\beta \ll 1$) while $Ma \sim e^{-c\beta}$ for $\beta \to \infty$ in asymptotically free theories.

In principle we should examine Ma and other quantities for signs of phase transitions, such as discontinuities or other non-analytic changes of behaviour, as β increases from zero to ∞. If there is a transition, we need to examine $E(R)$ in the asymptotically free phase to see whether it is confining. In practice we cannot take $a \to 0$, $\beta \to \infty$ in either strong-coupling or numerical calculations but it is sufficient to go far enough to observe the behaviour predicted by asymptotic freedom so that we can be sure that the $\beta \to \infty$ phase has been reached.

The leading-order strong-coupling result $g^2(a) \approx \frac{1}{3}\,e^{Ka^2}$ gives $g^2(a)/4\pi \approx 1$, 0.15 for $a = 1$ fm,

† This is for the Lagrangian (space–time lattice) formulation. In the Hamiltonian (space lattice) formulation, $g^2(a) \approx a^2$. This agrees with the intuitive expectation that, if $V \equiv g^2_{\text{eff}}(R)/R$, then for large R the behaviour of $g_{\text{eff}}(R)$ should be like the behaviour of $g(a)$ for large a so that a linear potential would correspond to $g^2(a) \approx a^2$. Although the Lagrangian and Hamiltonian gs behave very differently, the corresponding β functions only differ by $\ln g$ for $g \to \infty$.

0.7 fm respectively, so it is unlikely to be at all trustworthy for $a < 1$ fm. High-order calculations combined with clever extrapolation techniques are therefore needed to go to small a. A rapid but smooth transition from strong to weak coupling is found (Kogut *et al.* 1979; Münster 1980, 1981; Münster & Weisz 1980) but the conclusion that confinement and asymptotic freedom coexist is somewhat undermined by the existence of the roughening transition.† The alternative to strong-coupling expansions is to perform numerical calculations by using Monte Carlo techniques (Creutz 1980*a*, *b*; Wilson 1980). In a celebrated calculation Creutz (1980*a*, *b*) showed that in SU(2) and SU(3) Ka^2 departs abruptly but smoothly from strong-coupling behaviour at (for SU(3)) $\beta \approx 6$, $Ka^2 \approx 0.7$, $a \approx 0.4$ fm and exhibits weak coupling behaviour down to $Ka^2 \approx 0.07$, $a \approx 0.1$ fm, where the calculation breaks down. This is not definitive since K was fitted at $R = 3a$ which is not large in the weak-coupling region; it is conceivable that calculations with a lattice large enough to allow fits at $R \gg 1$ fm could reveal a phase transition at $\beta = \beta^*$ in the transition region $Ka^2 = O(0.7)$ and that the large-R potential is not confining for $\beta > \beta^*$. However, the action per 'plaquette' of area a^2 and its derivative (the specific heat in the statistical physics analogue) behave very smoothly in the transition region (Creutz 1980*a*, *b*; Lautrup & Nauenberg 1980; Edgar *et al.* 1981). Furthermore an SU(2) calculation in which $g(a)$ is determined by holding the action per unit physical area fixed as a is varied shows no evidence for a phase change and agrees well with the weak-coupling expansion from the transition region $\beta \approx 2$ to $\beta \approx 5$ where the calculation ends (Creutz 1981). There is therefore good evidence that asymptotic freedom coexists with confinement in SU(2) and SU(3).

The calculation of Ka^2 can be used to determine λ_{QCD} from K. The result (Creutz 1980*a*, *b*), when translated into continuum language (Hasenfratz & Hasenfratz 1980), is

$$\lambda^{\mathrm{mom}} = 170 \pm 50\,\mathrm{MeV},$$

in agreement with values obtained phenomenologically (and with the results of strong-coupling expansions (Kogut *et al.* 1979; Münster 1980, 1981; Münster & Weisz 1980)). Furthermore, the plaquette–plaquette correlation function yields $O(1.5)$ GeV for the mass of the lightest glueball (Berg 1980; Bhanot & Rebbi 1981), in agreement with phenomenological guesses, which is non-trivial given the small value of the input scale of $\sqrt{K}$.

Other theories have also been studied numerically, some of the most notable results being:

(i) Convincing evidence for a phase transition in U(1) (Creutz 1981*a*), including a demonstration that $E(R)$ is not confining for $\beta > \beta^*$ (Moriarty 1981*a*, *b*). The existence of a phase transition can be proved analytically (Guth 1980), the non-confining weak coupling phase become continuum QED for $a \to 0$.

(ii) The discovery of a phase transition in SO(3) (Halliday & Schwimmer 1981; Greensite & Lautrup 1981). Continuum SO(3) coincides with SU(2) so this phase transition is not decon-

† When Q and $\bar{\mathrm{Q}}$ lie on a common line through the lattice the flux lines follow this line for $\beta \ll 1$. However, in the continuum, a string of flux can experience transverse fluctuations with zero expenditure of energy for $\lambda \to \infty$, and the root-mean-square transverse displacement $\sigma(R)$ of a string of length R presumably behaves like $\ln R$. There is therefore a 'roughening' transition from $\sigma = 0$ to $\sigma \sim \ln R$ as $a \to 0$. Presumably it is not deconfining. However, it introduces non-analytic behaviour which prevents the extrapolation of strong-coupling results to small a and will slow down the convergence of numerical calculations near the transition. In the Hamiltonian formulation of the strong-coupling expansion the roughening transition occurs in the region where weak coupling behaviour is already observed and therefore does not spoil the inference that asymptotic freedom and confinement coexist (Kogut *et al.* 1981*a*). In fact the roughening transition can be avoided by placing $\bar{\mathrm{Q}}$ 'off axis' relative to Q so that straight flux lines are impossible (Kogut *et al.* 1981*b*).

fining. We are always free to choose the most convenient lattice Lagrangian corresponding to a given continuum theory, for example the SU(2) lattice Lagrangian in this case. For a discussion of the effect of various changes of the Lagrangian see Lang *et al.* (1981).

(iii) The discovery of a phase transition in SU(4) and SU(5) (Creutz 1981 *b*; Bohr & Moriarty 1981 *c*). Creutz suggests that this transition is not deconfining and might be avoided by changing the lattice Lagrangian (if it is deconfining it would signal trouble for the $1/N$-expansion).

The greatest challenge facing lattice QCD at present is to include light fermions, which could conceivably alter the phase structure since they have a deconfining shielding effect. It seems to be accepted orthodoxy that light quarks will not change the general features of the heavy quark–gluon sector of the theory but this is far from obvious, especially since $\langle\bar{q}q\rangle \neq 0$. Future problems include investigation of chiral symmetry-breaking for the $I = 1$ and $I = 0$ currents (for a recent investigation of a single-quark theory in the large-N limit and references see Kluberg–Stern 1981). We could hope eventually to obtain many accurate quantitative results but this may require breakthroughs allowing bigger lattices in numerical calculations or new analytic techniques.

6. Unconfined quarks?

Fairbank and collaborators (La Rue *et al.* 1981) have recently reported evidence for the existence of objects with charge $\frac{1}{3}e$ which could contain free quarks. Unless there is some quite unsuspected flaw in the lattice calculations, QCD is incompatible with unconfined quarks unless light quarks alter the phase structure, producing a non-confining asymptotically free phase with the boundary for $a \to \infty$ at large g. Presumably there would have to be a breakdown of local $SU(3)_c$, leaving global $SU(3)_c$ intact, to give mass to the unconfined gluons.

There are two possibilities for the q–$\bar{q}$ potential.

(i) It could increase to a large constant value at large R. Free quarks would then be very heavy.

(ii) It could increase to a large value and then decrease to a lower plateau asymptotically. Free quarks could then be quite light.

In either case quarks would have a large 'appetite' to swallow nucleons, the dipole quark–nucleon force presumably being stronger than the 'Van der Waals' nucleon–nucleon force, but it would be harder to indulge this appetite in case (ii). Quarks could be large objects, able to create virtual excitations in the region $\mathrm{d}E(R)/\mathrm{d}R > 0$, and the cross section to produce them would be small. De Rujula *et al.* (1978) have investigated a bag-based model of type (i) but it is unclear to what extent their specific results depend on the bag boundary condition $\boldsymbol{E}_c \cdot \boldsymbol{n} = 0$ on the surface, originally introduced to enforce confinement, which fails here because the gluons have acquired mass by the Higgs mechanism so that $\int \boldsymbol{E}_c \cdot \mathrm{d}\boldsymbol{s} \neq \rho_c$.

Further quark searches are clearly needed. If they support the Fairbank experiment the theory of unconfined quarks will rapidly get the attention needed to bring it into a respectable state.

7. Conclusions

QCD is the only field theory compatible with global colour symmetry and chiral symmetry. It provides a qualitative description of the short- and long-distance properties of hadrons. At present QCD faces no glaring paradoxes or puzzles, apart perhaps from the possible existence of free quarks.

The major issue for the future is to perform lattice calculations which include fermions or to find some other way to discover whether QCD really leads to (quasi?) confinement, spontaneous chiral symmetry-breaking and all the other conspicuous features of hadrons.

Note added in proof, 6 *October* 1981. Very recently Bodwin *et al.* (G. T. Bodwin, S. J. Brodsky & G. P. Lepage SLAC-PUB 2787 August 1981) have claimed that soft gluon effects modify factorization for processes with more than one hadron in the initial state or one initial hadron and at least one detected hadron. They assert that in these processes soft gluon exchanges between the 'hard' partons and the 'spectator' partons make a leading twist contribution (in order α_s^2) which can change the colour but not the momentum of the hard partons. The only effect this would have on the results obtained by assuming exact factorization is to change the normalization by an unknown amount. In pp $\rightarrow \mu\bar{\mu}X$ it would tend to wipe out the N_c^{-1} factor and increase the cross section. Detailed calculations will be needed to substantiate this claim. Bodwin *et al.* explain why with nuclear targets the leading twist contributions to $\langle \vec{p}_T^2 \rangle$ are A independent but point out that higher twist effects will depend on A, e.g. in pp $\rightarrow \mu\bar{\mu}X$ there will be an energy-independent contribution proportional to $A^{\frac{1}{3}}$.

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